

Content packages for interactive whiteboards: Combinatorics, statistics, and probabilities

This guideline provides an overview of the content and didactic context of the media in the content packages for interactive whiteboards entitled “Combinatorics”, “Probabilities,” and “Statistics”. General information on the use and teaching concept of the content packages for interactive whiteboards is provided in the teaching method “Working with a content package for interactive whiteboards,” which is also included in the media package.

1 Introduction to teaching this topic

1.1 Motivation for the topic

The topic of stochastics with its branches “probabilities,” “statistics,” and “combinatorics” has long been covered at elementary schools in countries like Germany or Japan. In Germany, for example, the “gathering and representation of information in graphics, tables and charts” is specifically stipulated in the mandatory curriculum for applied mathematics in all four grades of elementary school, and the solving of “combinatorial problems” is also explicitly required. The actual educational standards for content-related areas of expertise include the systematic counting of possibilities (combinatorics), the estimation and comparison of chances of winning in random experiments (probability), as well as the acquisition of data and frequencies of occurrence (statistics) in observations, investigations and experiments and the presentation of such data in tables, charts and diagrams. This in turn stimulates and further develops mathematical abilities such as understanding the principle of counting, ability to solve problems and logical thinking. The intermeshing of the three levels enactively (acting), iconically (in visual form) and symbolically is extremely important when dealing with the topics of stochastics or combinatorics at elementary school. For this reason, the following materials and examples have been selected so that they are familiar to students from their everyday life, and will motivate them and stimulate them to act (en-active level). After visual illustration of the situation, it is hoped that children will be encouraged to consider their own notation possibilities (symbolic level) from their own experiences and to compare them.

1.2 Media selection

There are a large number of media for use in connection with the topics of stochastics and combinatorics. If we were to list them all in a single content package for interactive whiteboards this would be at the expense of clarity. That is why the media have been grouped according to the three branches Statistics, Combinatorics and Probabilities and spread over three “smaller” content packages. These content packages contain the following media:

Content package for interactive whiteboards: “Statistics”

- 1 photo (cover image) for illustrating relevance to everyday life
- 1 graphic on the topic of acquisition of statistical data
- 1 interactive graphic on the acquisition and representation of statistical data
- 4 printable worksheets (one with answer sheet) with assignments on statistics
- 1 link list with links to other information on the topic

Content package for interactive whiteboards: “Combinatorics”

- 1 photo (cover image) for illustrating relevance to everyday life
- 1 graphic on the topic of number combinations
- interactive graphics for displaying combinatorial results in the form of drawing, table and tree diagram
- printable worksheets with assignments on combinatorics (with answer sheets)
- 1 link list with links to other information on the subject.

Content package for interactive whiteboards: “Probabilities”

- 1 photo (cover image) for illustrating relevance to everyday life
- 1 interactive graphic on the probabilities of the results of rolling dice
- interactive exercises (cloze test and matching exercise)
- experimentation instructions: Random sample experiment (with answer sheet) and dice experiment
- 1 printable worksheet with assignments on probabilities (with answer sheet)
- link list with links to other information on the subject.

1.3 Background information for teachers

The media files of the content packages for interactive whiteboards can be used individually and independently of each other. But the lessons will surely be livelier when the media are used in combination. To that end, we recommend working through the three subtopics in the following steps. The assignments given as examples in each case for working on in class are suitable for all grades, whereas some of the exercises for individual study are divided into two levels of difficulty (grades 1-2 and 3-4).

Statistics

- Extraction and comparison of information from a chart
- Acquisition of data and frequencies and graphical representation of the results
- Various problem-solving assignments involving statistics

Combinatorics

- Graphical representation of combinatorial problems
- Representation of combinatorial problems in diagrams
- Representation of combinatorial problems in tables
- Various problem-solving assignments involving combinatorics

Probability

- Familiarization with the terminology
- Guessing and testing of probabilities
- Estimation, checking, and recording of chances of winning
- Various problem-solving assignments involving probability

It should be mentioned at this point that the media contained in the content packages for interactive whiteboards cannot offer a complete teaching sequence, but must be supplemented by the teacher with further practical examples and exercises.

The teacher should also select suitable social and work forms in advance that correspond to the children's level of knowledge.

Tip: Mathematics conferences in which students “invent” and discuss possible solutions in groups in advance are appropriate with virtually all media.

Links to further information on each of the three subtopics are compiled in a link list:

Media



“Link list to the ‘Statistics’ interactive whiteboard content



“Link list to the ‘Combinatorics’ interactive whiteboard content “



“Link list to the ‘Probabilities’ interactive whiteboard content”

2 Content package for interactive whiteboards: “Statistics”

Statistics is the collection of data and frequencies through survey or observation and the representation of such data in diagrams or tables.

2.1 Extraction and comparison of information from a chart

Assignment

What are the most popular hobbies among children?

Objectives

- Reading simple diagrams
- Extracting information
- Forming concepts (more/less than, equal to, least/most)
- Determining and representing data

Implementation

The students are required to read off, compare, and verbalize numbers from a bar chart that can be built up interactively.

Medium



“What are the most popular hobbies among children?”

Students can enter the data from the bar chart in a table on the corresponding worksheet. Statements on the contents of the bar chart can then be assessed as right or wrong.

Medium



“Popular hobbies among children” (with answer sheet)

Tip: As an exercise, students can ask their classmates about their hobbies and draw up a chart themselves.

2.2 Acquisition of data and frequencies and graphical representation of the results

Assignment 1

What pets do the children in our class have?

Objectives

- Collecting data and quantities
- Representing in tables and diagrams
- Forming concepts (more/less than, equal to, least/most)

Implementation

For the assignments in the worksheet, students collect data themselves in class in order to apply the knowledge learned, in this case using the example “Pets.” The results are first recorded in a table in the form of a tally sheet and then represented in a bar diagram. Subsequently, students have to formulate questions on the results of the data survey, which are then answered in class.

Medium



“What pets do the children in our class have?”

Assignment 2

Look for any data or quantities in the picture that you can represent and compare in tables.

Objectives

- Collecting data and quantities
- Representing in tables and diagrams
- Forming concepts (more/less than, equal to, least/most)

Implementation

The students are required to extract information from a picture (numbers of plants, animals, etc.), and to represent and compare it in order to apply the knowledge learned, this time working on their own.

Medium



“Landscape with plants and animals”

2.3 Various problem-solving assignments involving statistics

These worksheets in two levels of difficulty (grades 1-2 and 3-4) with an additional assignment in each case for especially “fast” students offer suggestions for topics on which the children can collect and graphically represent data and frequencies in class or in the school.

Media



“Collecting and representing data” (grades 1-2)”



“Collecting and representing data” (grades 3-4)”

An answer sheet cannot be provided in this case because the data will vary from class to class. It is therefore important to check the prepared charts together with the students and in particular to verbalize the results using the terms they have learned.

3 Content package for interactive whiteboards: “Combinatorics”

Combinatorics means the systematic counting of possibilities. The results can be shown in graphical form (in drawings or diagrams) or in tables. The number of possible combinations is then read off from there. The solutions can also be calculated mathematically.

3.1 Graphical representation of combinatorial problems

Assignment

Three friends want to be in a picture together – What possible combinations do they have of arranging themselves?

Objectives

- Understanding and verbalizing situations with combinatorial content
- Preparing simple drawings

Implementation

Before the students show the different arrangement possibilities in a graphic, they should physically experience the situation and try out the possibilities (enactive levels). Three children arrange themselves in different orders and verbalize who is standing where.

The graphical representation of combinatorial problems is now worked out interactively by dragging drawings of three children in arbitrary combinations into “picture frames” (iconic level):

Medium



“Combining faces for pictures (drawing)”

Information: One correct combination can be shown by clicking the corresponding button.

The students are then required to find other, simpler illustrative notation possibilities for this assignment (stick figures, letters of the alphabet, etc.), in order to discover the clearest variant for them to use later in solving similar assignments (symbolic level).

3.2 Representation of combinatorial problems in diagrams

Assignment

Thomas has three pairs of pants and three T-shirts in different colors – What possible ways are there for him to dress?

Objectives

- Understanding and verbalizing actual situations with combinatorial content
- Representing in a tree diagram

Implementation

Before the students represent the various different possible combinations in a diagram, they should physically experience the situation and try out combinations. Real clothing or doll’s clothing is therefore combined together in different ways and the results verbalized (enactive level).

The class now works through the representation of the results in a tree diagram interactively. The children will later prepare diagrams on their own in similar assignments:

Medium



“Combining pants and T-shirts (tree diagram)”

Information: One correct combination can be shown by clicking the corresponding button.
The students will then look for possible ways of making the diagram simpler, for example, by using abbreviations (symbolic level).

3.3 Representation of combinatorial problems in tables

Assignment

Thomas has three pairs of pants and three T-shirts in different colors – What possible ways are there for him to dress?

Objectives

- Understanding and verbalizing situations with combinatorial content
- Showing results in a table
- Calculating the solution

Implementation

In this graphic, students can drag graphical symbols of pants and T-shirts in any combinations into a table with two columns. They must then verbalize the combinations they have found:

Medium



“Combining pants and T-shirts (table)”

Information: One correct combination can be shown by clicking the corresponding button.
The next step is to work out the mathematical solution ($3 \times 3 = 9$). The students do not have to be able to apply this method of calculation, although it may be helpful for some children for finding the solution or it can be used for checking.

3.4 Various problem-solving assignments involving combinatorics

The students are required to find three-digit combinations for a bicycle lock by trial and error. In addition, there are five-digit combinations for fast students. For this purpose, they will use the numerical cards 1 – 9 (cut-out sheet contained in the worksheet). They should then think about the number of possible combinations. Using the answer sheet the solution can be worked out systematically in class by means of calculation and by entering the results in a diagram as a way of applying the knowledge learned:

Medium



“Combine numbers” (with answer sheet)

The graphic with the combination locks from the worksheet is also available as an individual image and can be filled in, for example, by hand on the interactive whiteboard.

Medium



“Possible combinations of a combination lock”

The following two worksheets (two levels of difficulty: grades 1-2 and 3-4), each with an answer sheet, offer various exercises on the topic of combinatorics. In grades 1-2 the teacher specifies one of the three representation forms to use, while in grades 3-4 it is left up to the students themselves to select the most practical one for their purposes:

Media



“Find the possible combinations (grades 1-2)” (with answer sheet)



“Find the possible combinations (grades 3-4)” (with answer sheet)

4 Content package for interactive whiteboards: “Probability”

Probability is a classification of statements and judgments according to the degree of certainty. The occurrence of an event can accordingly be certain, possible, impossible, likely, or unlikely. Random experiments can be conducted to determine the probability and their results shown in tally sheets or tables.

4.1 Familiarization with the terminology

Assignment

Fill in the blanks with the terms certain, possible, impossible, likely, or unlikely.

Objectives

- First experiences relating to probability
- Forming concepts (certain, possible, impossible, likely, or unlikely)

Implementation

In this cloze test, the terms certain, possible, impossible, likely, or unlikely are selected with reference to everyday situations.

Medium



“Probabilities in everyday life (cloze test)”

4.2 Guessing and testing of probabilities

Assignment

Roll two dice 50 times. What sum total occurs most frequently?

Objectives

- Experiences relating to probability
- Forming concepts (certain, possible, impossible, likely, or unlikely)
- Representing the results of a random experiment in a table

Implementation

Before conducting the random experiment, the students should try to guess what total values are possible and how high the degree of probability is. The results are recorded on a tally sheet, and the terms learned by the students are inserted in a cloze test depending on the frequency of the values:

Medium



“Probabilities: Experiment with dice”

Following the experiment, all possible combinations of dice will be worked out in class. If some students have already found them out for themselves (see additional assignment in the experimentation instructions), they can present their results.

An interactive graphic showing the possible combinations of dice for selected dice totals can be used to summarize the knowledge learned or as an alternative to the experiment for working through the topic:

Medium



“Dice totals”

There is also an interactive matching exercise for further study with additional questions on the terms learned and the possibilities:

Medium



“Frequencies of dice totals (matching exercise)”

Note: This interactive assignment also serves as the answer sheet for the cloze test in the experimentation instructions.

4.3 Estimating, checking, and recording of chances of winning

Assignment

Pull a token out of a cup

Objectives

- Estimating and checking chances of winning in random experiments
- Forming concepts (certain, possible, impossible, likely, or unlikely)
- Representing the results in a table

Implementation

Together in groups, the children are asked to pull different colored tokens from cups and to record the results in a table. Then they compare the results of the five cups and suggest ideas why the results are different. These ideas will be checked on the basis of the contents of the cups filled with different numbers of red and blue tokens. The findings are then verbalized in a cloze test (answer sheet provided) using the terms that have been learned:

Medium



“Random sample experiment” (with answer sheet)

There are a few instructions for teachers for preparing this experiment:

Medium



“Random sample experiment” (teacher information)

4.4 Various problem-solving assignments involving probability

Experiments and problems (incl. two additional assignments for fast students) relating to probability must be solved on a worksheet. An answer sheet is used for checking and should be discussed in class in order to formulate reasons for the results.

Medium



“What is certain, possible, impossible?” (with answer sheet)