Heat capacity

Definition

To heat a substance, that is, to bring it from a lower temperature to a higher temperature, you must add heat (Q) to it. The greater the temperature difference (ΔT) that is to be achieved, the more heat you must add; it is said that the two variables are directly proportional to each other:

$$Q \sim \Delta T \qquad (1)$$

The precise amount of heat that you must use in the process depends on the substance‘s properties and its size, i.e., its mass. The ratio of the amount of added heat to the resulting temperature difference is called the **heat capacity (C)**; in relation to the mass (m) of the substance, people refer to the **specific heat capacity (c = C/m)**. (The latter is often also referred to as the “specific heat” for short, which is ambiguous.)
(Note: The specific heat capacity itself depends on the temperature, and for gaseous substances, it also depends on the pressure and volume.)

If the heat capacity is known, the required amount of heat can be calculated using the following formula:

$$Q = C \cdot \Delta T = c \cdot m \cdot \Delta T \qquad (2)$$

The heat capacity of water

To heat 1 kg of water at 20 °C by 1 °C, you must add 4.183 kJ of energy (heat). As a comparison with the heat capacities of other substances shows, liquid water has the highest specific heat capacity. For instance, if you want to increase the temperature of 1 kg of copper at 20 °C by 1 °C, you need “only” 0.38 kJ of heat.

Substance	Heat capacity [kJ/kg K]
Water	**4.183**
Ethanol	2.4
Wood	2.5
Ice (0 °C)	2.1
Ammonia	2.06
Air	1.01
Aluminum	0.9
Copper	0.38
Mercury	0.14

Table 1: Standard values for the heat capacities of liquids, gases, and solids. Unless indicated explicitly, the values refer to a temperature of 20 °C. The data for gases are the heat capacities at constant pressure. (Source: Kuchling, 1978)

Why does liquid water have such a high heat capacity?
You can derive the reason for the high heat capacity of liquid water using the particle model as an illustration: The particles in the particle model have multiple independent possibilities for motion: translation, rotation, and vibration. (In technical terms, these independent possibilities for motion are referred to as “degrees of freedom.”) All of these different motions can be induced through the addition of heat. The higher the number of different motions that can be induced, the higher the heat capacity is.
Liquid water has a particularly high number of degrees of freedom, since the water molecules are angled (several degrees of freedom with respect to rotation) and, in addition to their natural vibrations, the water molecules can also produce vibrations against each other. Collectively, these many degrees of freedom result in the high heat capacity of liquid water.

How to determine the heat capacity of a substance

First, determine the temperature and the mass of the substance whose heat capacity you want to determine. Then add a defined amount of heat to the substance, measure the temperature again, and determine the temperature difference (ΔT). Then calculate the specific heat capacity from the measured variables using the following formula:

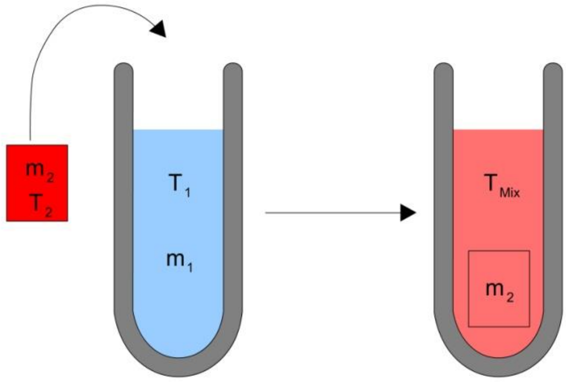
$$c = Q / (m \cdot \Delta T) \qquad (3)$$

Liquids or gases
Determining the heat capacity experimentally is quite simple, as an example from the kitchen shows.
Boil water: Heat one liter of water (mass = 1 kg) for three minutes in an electric kettle with 2,000 watts of electrical power. You can calculate the amount of heat added based on the power and time ($Q = P \cdot t$), you can measure the water’s temperature when you fill the electric kettle and after heating the water, and then you have all the variables you need to calculate the heat capacity.
(Note: Following this method, you are making a fairly big error in the measurement. This is because heat is lost to the environment and the electric kettle’s container absorbs heat. If you want to determine the water’s specific heat capacity precisely, you must work with an extremely well-insulated container and know its own heat capacity.)

Solids
You determine the heat capacity of solids using a **calorimeter**, which is based on the law of mixtures (see equation 4):
If you bring two solid bodies with different temperatures into contact with each other, heat transfers from the warmer solid body (m_1, T_1, c_1) to the cooler body (m_2, T_2, c_2) until the two solid bodies have the same temperature (mixed temperature T_{Mix}). The amount of heat given off by the warmer body (Q_1) is equal to the amount of heat absorbed by the cooler solid body (Q_2).

$$Q_1 = c_1 \cdot m_1 \cdot (T_1 - T_{Mix}) = c_2 \cdot m_2 \cdot (T_{Mix} - T_2) = Q_2 \qquad (4)$$

When you use a calorimeter, you bring the substance to be measured (m_2, T_2) into contact with a water bath (m_1, T_1). It is important to use a well-insulated container to hold the water so that little heat is lost to the environment. The heat capacity, mass, and temperature of the water are known. Now heat the water for a certain amount of time. When you stop adding heat, measure the water's temperature.



If you now place a solid body into the water, you will notice that the water bath cools down, because a certain amount of the heat is transferred to the solid body in the water bath. As soon as the temperature stops changing, the water and the solid body are in thermal equilibrium. The mixed temperature T_{Mix} has been reached, and you can calculate the solid's heat capacity by solving the equation (4) for the heat capacity of the cooler substance (c_2).

$$c_2 = c_1 \cdot m_1 \cdot (T_1 - T_{Mix}) / m_2 \cdot (T_{Mix} - T_2) \qquad (5a)$$

In this case as well, you are making the error of not taking into account the heat capacities of the calorimeter and the thermometer (added together referred to as C_K). If you want to be precise, you must first determine this heat capacity (C_K) using water and then take it into account when calculating the solid's specific heat capacity (c_2). The formula then looks as follows:

$$c_2 = (c_1 \cdot m_1 + C_K) \cdot (T_1 - T_{Mix}) / m_2 \cdot (T_{Mix} - T_2) \qquad (5b)$$

Heat capacity and heat storage

Energy storage is an important topic in connection with the use of renewable energies to generate electric power. One option to store energy is in the form of heat.
Everyone is familiar with heat storage based on an example from everyday life. In winter, people like to fill a hot-water bottle to keep them warm and cozy throughout the night. In this case, the heat is "buffered" in the form of a temperature increase. For this type of heat storage, substances with a higher heat capacity such as water provide a particular advantage: The higher the heat capacity, the less mass of the substance is needed to store a certain amount of heat in it (after all, hot-water bottles are generally not very large).
Note: When energy is stored in the form of heat on an industrial scale, the magnitude of the substance's heat capacity is of secondary importance. Other types of heat stores are preferred, for example, latent heat stores (in this case, heat is stored in the form of a phase transition) or chemical heat stores (heat is stored in the form of chemical reactions). For example, when a sodium acetate store with a mass of 100 kg is melted at 58 °C, it holds five times the amount of heat of a water store with a mass of 100 kg that is heated from 0 °C to 99 °C.